yu_*/ν, $\delta^+ = \delta u_*/\nu$, $U^+ = U/u_*$, $\theta^+ = \theta u_*^2/\nu$, $\Delta_\nu^+ = \Delta_\nu u_*/\nu$, $\Delta_\theta^+ = \Delta_\theta u_*/\nu$, $\Delta_\alpha^+ = \Delta_\alpha u_*/\nu$, dimensionless quantities in the near-wall variables; A, K, B_0, universal constants in the logarithmic and root-mean velocity distributions; c_1, c_2, c_3, numerical coefficients; u_{*cr}, stress at which the influence of the polymer on the mean velocity profile starts; $\alpha(\pi)$, parameter characterizing the influence of the polymer; α_{cr}, minimal pressure gradient at which the influence of the pressure on the boundary layer starts; c_f, local friction coefficient.

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HEAT EXCHANGE DURING FLOW OF ANOMALOUSLY VISCOUS FLUIDS IN CYLINDRICAL CHANNELS OF SIMPLY CONNECTED CROSS SECTION

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UDC 532.517.2

A method is proposed and results of a numerical solution are presented for a problem of heat exchange on the initial section of cylindrical channels of simply connected cross section during steady-state flow of an anomalously viscous fluid.

A theoretical investigation of heat exchange during flow of an anomalously viscous fluid in cylindrical channels of simply connected cross section has great applied importance.

A considerable number of studies [1, 2] have been devoted to questions of the heat exchange of anomalously viscous media for their flow in prismatic simply connected channels. However, in connection with the fact that the treatment of the given question encounters large mathematical difficulties, the known studies have either been of an experimental nature or have been devoted to a consideration of particular cases (a "power" rheological law, flow in channels of simplest forms, etc.). A fundamental obstacle for calculating the heat exchange in prismatic channels is the absence of analytical methods of determining the velocity profile in an anomalously viscous medium.

The aim of the present study is to solve the problem of heat exchange on the initial section of a cylindrical simply connected channel for flow of an anomalously viscous fluid described by an arbitrary rheological law for the case of boundary conditions of the first kind.

Considering laminar steady-state flow of an anomalously viscous fluid in a prismatic channel for the condition that heat transfer owing to heat conduction along the axis of the channel is incommensurably small in comparison with the forced transfer and dissipative release of heat is insignificant, the problem can be formulated in the following way:

$$v(x,\ y)\frac{\partial U}{\partial z} = a\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right), \tag{1}$$

S. M. Kirov Kazan' Chemicotechnological Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 6, pp. 988-993, December, 1979. Original article submitted February 5, 1979.

0022-0841/79/3706-1401$07.50

$$x,\ y \in G = \bar{G} + \Gamma,\ 0 \leqslant z \leqslant Z,$$
$$a = \text{const}.$$

The boundary conditions of the first kind are written as

$$U(x,\ y,\ 0) = U_0,\ U(x,\ y,\ z)|_\Gamma = U_\Gamma. \tag{2}$$

The velocity profile can be found from

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy \tag{3}$$

by means of the integration

$$v(x,\ y) = \int \varphi \left(\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy \right) + C, \tag{4}$$

where the fluidity φ is expressed by the function

$$\varphi = f(\tau^2) = f(\tau_x^2 + \tau_y^2) = f\left\{(0.5\Delta P)^2 \left[\left(\frac{\partial \psi}{\partial x} \right)^2 + \left(\frac{\partial \psi}{\partial y} \right)^2 \right] \right\}. \tag{5}$$

The constant of integration in (4) equals the maximum value of the velocity v_{max}.

The method of determining the velocity profile that is used was obtained under the assumption that the nature of the shear stress distribution over the cross section of the channel in an anomalously viscous medium is similar to the case of the flow of a viscous fluid under the same conditions, and we apply it accordingly for anomalously viscous systems that do not have noticeable elastic properties [3].

Equation (4) describes the velocity distribution in channels with such a form of cross section, for which we know the solution of the Dirichlet problem for ψ in the Poisson equation. At the present time, for finding the function ψ and the solution of the indicated problem we have developed a reliable mathematical apparatus; with the help of this calculation we add channels of the same arbitrary form of cross section. For a number of complex profiles of channels the expression for the function ψ is obtained in the form of special functions, series, etc., and has a cumbersome form. Therefore direct integration of Eq. (4) for complex profiles can be sufficiently easily realized numerically using a computer.

In connection with this, the formulated problem of heat exchange was solved numerically on a computer, with derivation of Eq. (4) on a separate block. Such a formulation of the problem enabled us to develop a typical program and block diagram of solution for channels of arbitrary cross section.

Using the notation of [4], we write the system (1), (2) in the form

$$\frac{v}{a} \neq \frac{\partial U}{\partial z} = LU,\quad L = \sum_{\beta=1}^{2} L_\beta, \tag{6}$$

$$L_\beta U = \frac{\partial^2 U}{\partial \chi_\beta^2},\ \chi \in \bar{G},\ 0 \leqslant z \leqslant Z,$$

$$U(\chi,\ z)|_\Gamma = U_\Gamma,\ U(\chi,\ 0) = U_0,\ \chi = (\chi_1,\ \chi_2). \tag{7}$$

In the region $\bar{G}$ being considered the following conditions are satisfied: 1) intersection of the region $\bar{G}$ with any line parallel to the coordinate axes consists of a finite number of intervals; 2) it is possible to construct in the region G a connected grid $\bar{\omega}_h$ with step h_β, $\beta = 1, 2$.

The set ω_h of the interior nodes of the grid consists of the points $\chi = (\chi_1, \chi_2) \in G$ of intersection of the lines $\chi_\beta = i_\beta h_\beta$, $i_\beta = 0, \pm 1, \pm 2, \ldots$, $\beta = 1, 2$, the set γ_h of boundary nodes — from the points of intersection of the lines C_β, $\beta = 1, 2$, passing through all the interior nodes $\chi \in \omega_h$ with boundary Γ. For a difference approximation of the operator L_β at the node χ we chose a three-point pattern consisting of the points $\chi^{(-1\beta)}$, χ, and $\chi^{(+1\beta)}$. The difference operator $\Lambda_\beta \sim L_\beta$ has the form: a) at regular nodes

$$\Lambda_\beta Y = Y_{\bar{\chi}_\beta \hat{\chi}_\beta} = \frac{1}{h_\beta^2} (Y^{(+1\beta)} - 2Y + Y^{(-1\beta)}), \tag{8}$$

b) at nonregular nodes

$$\Lambda_\beta Y = Y_{\bar{\chi}_\beta \hat{\chi}_\beta} = \begin{cases} \dfrac{1}{h_\beta}\left(\dfrac{Y^{(+1\beta)} - Y}{h_\beta} - \dfrac{Y - Y^{(-1\beta)}}{h_\beta^*}\right), \ \chi^{(-1\beta)} \in \gamma_{h,\beta}, \\ \dfrac{1}{h_\beta}\left(\dfrac{Y^{(+1\beta)} - Y}{h_\beta^*} - \dfrac{Y - Y^{(-1\beta)}}{h_\beta}\right), \ \chi^{(+1\beta)} \in \gamma_{h,\beta}. \end{cases} \tag{9}$$

On the interval $0 \leqslant z \leqslant Z$ we introduce the grid $\bar{\omega}_{\tau^*} = \{z_j = j\tau^*, \ j = 0, 1, \ldots, j_0\}$ with step $\tau^* = z/j_0$.

Formally, replacing the polynomial heat equation by an array of one-dimensional heat equations, and approximating each equation of number β on the semiinterval $z_j + (\beta - 1)/2 \leqslant z \leqslant z_j + \beta/2$ by a two-layer scheme, we obtain two implicit locally one-dimensional schemes

$$\frac{Y^{j+\beta/2} - Y^{j+(\beta-1)/2}}{\tau^*} = \Lambda_\beta Y^{j+\beta/2}. \tag{10}$$

With each equation we associate a boundary condition

$$Y^{j+\beta/2} = U_\Gamma \ \text{ for } \ \chi \in \gamma_{h,\beta}, \tag{11}$$

$$Y(\chi, 0) = U_0.$$

For known Y^j we find the value of Y^{j+1} on a new layer from the solution of both of Eqs. (10) with boundary conditions (11), successively assuming β = 1, 2.

For finding $Y^{j+\beta/2}$ we obtain, according to [4], a boundary-value problem of the form

$$\begin{gathered} A_{i_\beta} Y_{i_\beta - 1}^{j+\beta/2} - C_{i_\beta} Y^{j+\beta/2} + B_{i_\beta} Y_{i_\beta + 1}^{j+\beta/2} = -F_{i_\beta}^{j+\beta/2}, \\ Y^{j+\beta/2} = U_\Gamma \ \text{ for } \ \chi \in \gamma_{h,\beta}, \ \beta = 1, 2, \end{gathered} \tag{12}$$

in which only the varying lower indices are indicated. The difference equation (12), written along the segment Δ_β lying on the line C_β, is solved by the pivotal method along all the segments Δ_β for fixed β. The function (4) in the region G is approximated by the function W(χ).

For solution of the system (1), (2) reduced to the boundary-value problem (12), we formulated a program in the language PL/1 for an EC-1020 computer. For convenience in calculation we carried out a transformation of coordinates — the coordinate origin was shifted in such a way that the region G was located completely in the first quadrant of a Cartesian coordinate system. Thus, the set ω_h can be represented in the form of a rectangular matrix of order p, n, where $p = \max(i_1 + 1)$ and $n = \max(i_2 + 1)$, with nonzero elements at the nodes belonging to $\bar{G}$.

Introducing the set $H_\beta = \{h_{i_\beta}^*, N_{i_\beta}', N_{i_\beta}''\}$, $i_\beta = 1, 2, 3, \ldots$, β = 1, 2, we can determine the nonzero elements of the matrix, and give initial and final values of the indices of the pivot coefficient of the difference Eq. (12). For solution of problem (12), we formulated matrices of the same order W, M_1, M_2, and M_3. The elements of the matrices M_1, M_2, and M_3 are, respectively, the values of the functions $Y^j(\chi)$, $Y^{i+1/2}(\chi)$, and $Y^{j+1}(\chi)$ in the region G.

At the beginning of the calculation, the matrix elements of M_1 are given the value $Y(\chi, 0) = U_0$. According to (10), for calculation of the matrix elements of M_2 the pivot was carried out in the direction of χ_1, and for M_3 — in the direction of χ_2, for which the set H_β is used. If the difference between the values of the elements of the matrices M_1 and M_3 is greater than the given ε, then the matrix element of M_1 are given the values of the matrix elements of M_3 and the elements of the matrix M_2 are calculated, etc. If this difference is less than the given ε, the calculation is stabilized (Fig. 1).

As an example we calculated the temperature fields for flow of an anomalously viscous fluid [4.75% aqueous solution of sodium carboxymethylcellulose (Na-CMC)] and a viscous fluid. In this case, as a specific rheological model of an anomalously viscous fluid we used the expression

$$\varphi = \varphi_0 + K\tau^m. \tag{13}$$

Figure 2 gives the distribution of the dimensionless temperature $\Theta = (U - U_\Gamma)/(U_0 - U_\Gamma)$, obtained as a result of the solution, along the axes of symmetry of an elliptical channel for various values of the relative length Z.

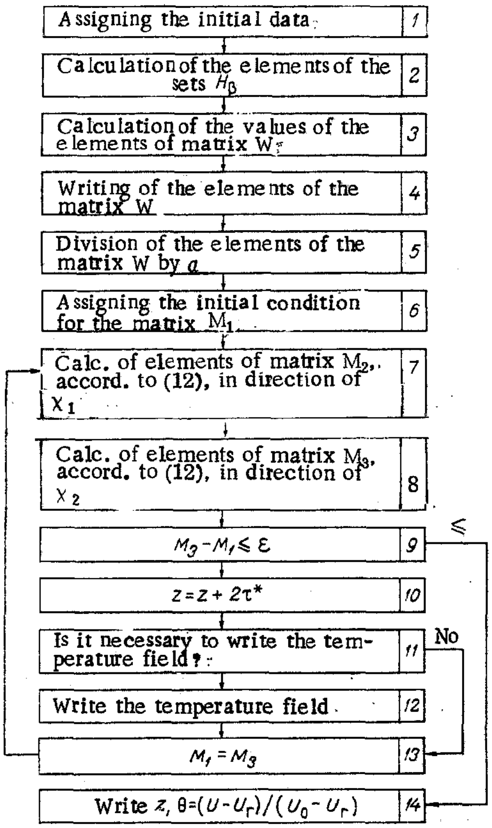

Fig. 1. Enlarged block diagram of the solution.

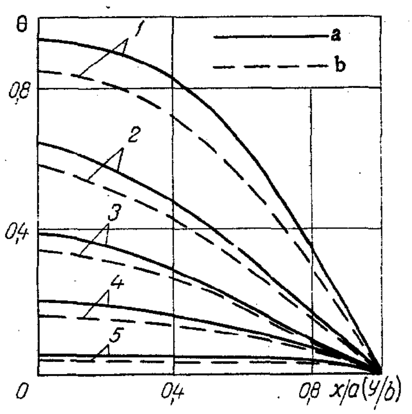

Fig. 2

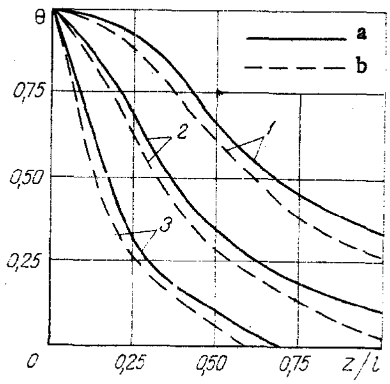

Fig. 3

Fig. 2. Distribution of dimensionless temperature Θ along the symmetry axes of the elliptical channel for flow: a) 4.75% solution of Na-CMC (K = 0.123; m = 0.35); b) viscous fluid (K = 0; m = 1) for U_0 = 20°C and U_Γ = 60°C (1 — z = 0.09; 2 — 0.25; 3 — 0.44; 4 — 0.61; 5 — 0.96).

Fig. 3. Variation of dimensionless temperature Θ along the length of the symmetry axis of the elliptical channel for flow: a) 4.75% solution of Na-CMC (K= 0.123; m = 0.35); b) viscous fluid (K = 0; m=1) for U_0 = 20°C and U_Γ = 60°C (1 — ΔP = 1200 N/m^2; 2 — 800; 3 — 400).

Figure 3 presents the variation of dimensionless temperature along the channel length for points of the center of symmetry of the ellipse for various hydrodymamic conditions of flow.

A comparison of the theoretically calculated and experimental values [5] of the mean coefficients of heat transfer for flow of 4.75% solution of Na-CMC in an elliptical channel showed that the maximum deviation between them is 20-23% and is due to the accuracy of describing the rheological equation (13) by a curve of the flow of a model anomalously viscous fluid.

NOTATION

v, flow velocity of the fluid; U, instantaneous temperature; x, y, z, moving coordinates; a, thermal conductivity of the fluid; Γ, contour of the channel; U_0, initial temperature of the fluid; U_Γ, temperature of the channel wall; Z, dimensionless length of the channel; Θ, dimensionless temperature; G, region; $\bar{G}$, region with boundary Γ; φ, fluidity of the fluid; τ, shear stress; τ_x and τ_y, shear stress components; ΔP, pressure differential per unit length of channel; ψ, a function that is a solution of the Dirichlet problem in the Poisson equation; $\chi = (\chi_1, \chi_2)$, a point of two-dimensional Euclidean space; h_β, step of the grid ω_h; γ_h, set of boundary nodes; C_β, a line passing through the interior nodes; ω_h, set of all regular nodes; $L_\beta U$, Laplacian operator; Λ_β, difference operator; h_β^*, distance from the nonregular node χ to the boundary node $\chi^{(+1\beta)}$ or $\chi^{(-\beta)}$; τ*, step of the grid along the z coordinate; $h_{i_\beta}^*$, distance from the near-boundary nodes $\omega_{h,\beta}^*$ to the boundary nodes $\gamma_{h,\beta}$; N_{i_β}', number of the left boundary nodes in the matrix in the direction of χ_β; N_{i_β}'', number of the right boundary nodes in the matrix in the direction of χ_β; φ_0, fluidity of the fluid for $\tau \to 0$; K and m, rheological constants.

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UNSTEADY HEAT TRANSFER IN A MICROPOLAR FLUID FLOWING IN A PLANE CHANNEL

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UDC 532.5:532.135

Heat transfer in a micropolar fluid flowing in a plane channel following an abrupt change in the wall temperature is investigated. The obtained results indicate that in several cases the fluid microstructure has a considerable effect on the main heat-transfer characteristics.

The theory of heat-conducting micropolar fluids (MPF) [1] can be used to characterize the hydrodynamic and thermal processes in several microstructural fluids (liquid crystals, suspensions, blood, etc.) with due consideration of the spinning of the particles in the medium. The hydrodynamics of MPF has now been widely investigated. There have been investigations of free convection, and also of steady heat transfer involving forced convection, where it was discovered that the microstructure of the fluid affects the characteristics of heat transfer in it. So far, however, due attention has not been paid to such an important practical problem as unsteady heat transfer in MPF.

We consider the following problem. A heat-conducting MPF flows between plane parallel plates separated by a distance 2h. Let the temperature of plates and MPF over the whole length of the channel be constant and equal to T_0. At a certain instant the temperature of the plates is abruptly altered and becomes equal to $T_j \neq T_0$. We determine the temperature field over the cross section and length of the channel in relation to time. We neglect energy dissipation, the compressibility of the MPF, axial heat conduction, and mass forces and moments. We regard the hydrodynamic velocity profile as stabilized, and the physical properties of the MPF as constant. The coordinate origin is on the central line at the entrance section of the channel, which has temperature T_0. The central line coincides with the

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 6, pp. 994-999, December, 1979. Original article submitted January 2, 1979.

0022-0841/79/3706-1405$07.50